Total No. of Printed Pages:05

SUBJECT CODE NO:- H-111
FACULTY OF SCIENCE AND TECHNOLOGY
S.E. (All Branches)
Engineering Mathematics - IV
(REVISED)

[Time: Three Hours] **[Max. Marks: 80]**

Please check whether you have got the right question paper.

N.B
1. Q. No. 1 and 6 are compulsory
2. Solve any two questions from the remaining questions of each section.
3. Figures to the right indicate full marks.
4. Assume suitable data, if necessary.

Section A

Q.1 Attempt any five 10

1. Find the Laplace transform of $e^{-t}\cos at$

2. Find the Laplace transform of $e^{-t}\frac{sint}{t}\delta(t-3)$.

3. Find the Laplace transform of $f(t)=\cos\left(t-\frac{2\pi}{3}\right), t>\frac{2\pi}{3}$

   $= 0, \quad t<\frac{2\pi}{3}$

4. Find the inverse Laplace transform of $\frac{s}{s^2a^2+b^2}$

5. Find the inverse Laplace transform of $\frac{s\,e^{-as}}{s^2+b^2}$

6. Form the partial differential equation by eliminating a and b from the equation

   $2z=\frac{x^2}{a^2}+\frac{y^2}{b^2}$

   OR

   Find z-transform of $\sin(\frac{k\pi}{2})$

7. Solve pq= 1

   OR

   Find the z-transform of $ka^k, k\geq 0$

8. Solve $2x\ \frac{\partial z}{\partial x} - 5y\frac{\partial z}{\partial y} = 0$

OR

Find the Z-transform of $e^k\ cos\alpha k, k \geq 0$

Q.2

a. Find Laplace transform of $e^{-4t}\int_0^t tsin3t\ dt$ 05

b. Find the inverse Laplace transform of $\cot^{-1}(s+1)$ 05

c. Find the solution of $\frac{\partial u}{\partial t} = h^2\frac{\partial^2 u}{\partial x^2}$ 05

using the conditions $u(o,t) = 0 = u(l,t), u\ (x,0) = \sin\left(\frac{\pi x}{l}\right)$

OR

find the z-transform of $\sin^2\left(\frac{k\pi}{4}\right), k \geq 0$

Q.3

a. Evaluate : $\int_0^\infty e^{-2t}t\sin ht\ dt$ 05

b. Find inverse Laplace transform by convolution theorem $\frac{s}{(s^2+a^2)(s^2+a^2)}$ 05

c. Solve $p\ tanx + q\ tany = tanz$ 05

OR

Find z-transform of $\frac{2^k}{k},\ k \geq 1$

Q.4

a. Express the following function in terms of Heaviside unit step function and hence find its Laplace transform 05

$f(t) = 2,\ 0 < t < \pi$

$= 0,\quad \pi < t < 2\pi$

$= sint, t > 2\pi$

b. Solve by Laplace transform $\frac{dy}{dt} + 2y + \int_0^t y\ dt = sint,\ y(0) = 1$ 05

c. Solve $p(1+q) = qz$ 05

OR

Find the inverse z-transform of $\frac{2Z}{(z-1)(z^2+1)}$ by residue method

Q.5 a. Find the Laplace transform of periodic function $f(t) = t, 0 < t < a$ 05
$= 2a - t, a < t < 2a$ and
$f(t) = f(t + 2a)$

b. Solve the following simultaneous L.D.E. by Laplace transform method 05

$$\frac{dx}{dt} - y = e^t, \frac{dy}{dt} + x = sint\ ,$$

Given $x(0) = 1,\ y(0) = 0$

c. Solve:$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ in the interval $0 \le x \le \pi$ subjected to the following conditions 05
$u(o, y) = 0 = 4(\pi, y),\ u(x, 0) = 1$
And $u(x, y) = 0\ as\ y \to \infty$

OR

Solve the difference equation by using Z-transform
$2y(k + 2) - 5y(k + 1) + 2y(k) = 0, k \ge 0$
Given $y(0) = 0,\ y(1) = 1$

Section B

Q.6 Attempt any five 10

a. Find the first approximate value of the root $(i.e.x_1)$ by Newton-Raphson method for $3x = \cos x + 1$

b. Find f(3) for the data

X :	1	2	4
f(x) :	14	15	5

c. Find the values of x ,y, z, in the first iteration by Gauss seidal method for
$2x + y + 6z = 9$
$8x + 3y + z = 24$
$2x + 17y + 4z = 35$

d. Find $\frac{dy}{dx}$ at x=1930 for the data

X:	1930	1940	1950	1960
Y:	40	60	79	103

e. Evaluate $\int_c e^z dz, where\ c: |z| = 1$

f. Find the image of $|z| = 2\ under\ w = \frac{1}{z}$

g. Find the poles and residues at each poles for $f(z) = \frac{Z^2}{(Z-1)(Z+2)}$

h. Find the values of A and B if $f(z) = X^2 + Ay^2 + iBxy$ is analytic

Q.7 a. Fit a straight line $y = a + bx$ to the following data by the method of least squares 05

X:	0	1	3	6	8
Y:	1	3	2	5	4

b. Solve by Gauss seidal method 05

$83x + 11y - 4z = 95$
$7x + 52y + 13z = 104$
$3x + 8y + 29z = 71$

c. Show that the function $w = \frac{4}{Z}$ transform the straight line x=a in the z-plane into a circle in w-plane 05

Q.8 a. Find $y'(o)$ and $y''(o)$ from the given table 05

X:	0	1	2	3	4	5
Y:	4	8	15	7	6	2

b. Show that $u = -sinx\ sinhy$ is harmonic and hence find its harmonic conjugate 05

c. Evaluate $\oint_c \frac{dz}{z^2 sinhz}$, where c is the circle $|Z - 1| = 2$ by cauchy's residue theorem 05

Q.9 a. Given that $\frac{dy}{dx} = \frac{y-x}{y+x}, y(o) = 1$ find y(0.1) by Euler's modified method. 05

b. If $f(z) = u + iv$ is an analytic function then find $f(z) if\ 2u + v = e^x(cosy - siny)$ 05

c. Evaluate $\oint_c \frac{z-3}{z^2+2z+5} dz$, where $c: |z+1-i| = 2$ by using cauchy's integral formula. 05

Q.10 a. Using Runge-kutta fourth order method find y(0.1)given that 05

$$\frac{dy}{dx} = 3x + \frac{y}{2}, \qquad y(o) = 1, \qquad take\ h = 0.1$$

b. Expand $f(z) = \frac{1}{(z+1)(z+3)}$ for $0 < |z-1| < 2$ by Laurent series 05

c. Evaluate $\int_0^{\pi-\pi i} e^{\bar{z}}\, dz$, along the curve x=t and y= -t 05